RYERSON UNIVERSITY

DEPARTMENT OF MATHEMATICS

MTH 210 FINAL EXAM - WINTER 2005

NAME: ____

STUDENT ID: ____

SECTION: ____

Section	Lab	TA	Section	Lab	TA
1	Thursdays at 1	Anca	3	Thursdays at 4	Chris
2	Thursdays at 5	Chris	4	Fridays at 9	Anca

INSTRUCTIONS

This exam has 6 pages including this front page. It consists of 3 parts and is worth 25% of the course mark. Please answer all questions directly on this exam.

This is a closed book exam. One 8.5" by 11" double-sided crib sheet is allowed, but no other aids are.

This exam is 3 hours long.

If you need more room for the solutions, please continue on the back of the page and indicate CLEARLY that you have done so.

Part A – Regular Expressions and Finite State Automata	20
Part B – Counting and Probabilities	20
Part C – Graph Theory	20
TOTAL	**60**

Part A – Regular Expressions and Finite State Automata – 20 Marks

Let L_1 be the language of the **binary representations** of all **positive** integers divisible by 4.
Let L_2 be the language of the **binary representations** of all **positive** integers **not** divisible by 4.
None of the elements of these languages have leading zeroes.

Please note that all the questions in this part of the exam are related, but it is possible to answer them independently. Therefore you can answer these questions in the order that makes the most sense to you.

A1 Regular Expressions

a) (2 marks) Write a regular expression denoting L_1.

b) (4 marks) Write a regular expression denoting L_2.

A2 Finite State Automata

a) (6 marks) Draw a state diagram (= deterministic finite state automaton) with as few states as possible which recognizes L_1. This state diagram should be complete: it should handle all strings of 0's and 1's.

b) (8 marks) Draw a state diagram (= deterministic finite state automaton) with as few states as possible which recognizes L_2. This state diagram should be complete: it should handle all strings of 0's and 1's.

Part B – Counting and Probabilities – 20 marks

B1 Final Exam Grades 6 marks

There are 116 students currently registered in mth210. For this question you should assume that this final exam is graded out of 50 (which is not actually true – see front page for real marking scheme) and that all the marks are integers.

a) What is the **least** number of final exams that will need to be graded to guarantee that at least 2 students in this class have the same grade in this final exam? Explain your answer.

b) You want to make a bet that there will be a group of at least x mth210 students who will all have the same grade on this exam. How large can you make x and still be **guaranteed** to win your bet? Explain your answer.

B2 Smart Hat (7 marks)

A hat contains 25 tokens of identical size, shape and weight. The tokens are numbered from 1 to 25. Your friend draws a token randomly from the hat and tells you that its number has two digits. What is the probability that your friend drew a token with a prime number? Explain your answer.

Note that the set of all prime numbers between 1 and 25 is {2, 3, 5, 7, 11, 13, 17, 19, 23}

B3 Treasure Chest

You have a drawer full of coins which include pennies, nickels, dimes, quarters, loonies, and toonies.

a) (3 marks) How many different collections of 5 coins can be made by picking 5 coins from that drawer? Explain your reasoning. You can leave your answer as a product of integers.

b) (4 marks) How many different collections of 5 coins (made by picking 5 coins from the drawer) contain at least 2 pennies? Explain your reasoning. You can leave your answer as a product of integers.

c) (Bonus marks) Each collection of 5 coins picked out of your drawer has a dollar value calculated by adding the values of the 5 coins. Is the number of possible dollar values of collections of 5 coins the same as the number of possible collections that you calculated in question a)? Explain your answer as precisely as possible.

Part C – Graph Theory – 20 marks

C1 Graph Isomorphisms 10 marks

a) For the pair of graphs below, completely justify why they are or are not isomorphic.

b) For the pair of graphs below, completely justify why they are or are not isomorphic.

C2 Graph Proof 10 marks

Prove that if a simple graph *G* is not connected, then its complement G' is connected.

Do **not** prove this by induction!

Definitions that you may find useful for this problem:

1) A **simple** graph is a graph that does not have any loops or parallel edges.
2) The **complement** *G'* of a simple graph *G* is the graph obtained as follows:
   - The vertex set of *G'* = the vertex set of *G*
   - Two distinct vertices *v* and *w* of *G'* are connected by an edge iff *v* and *w* are not connected by an edge in *G*.

   This definition can be written formally as:
   - $V(G') = V(G) = V$
   - $E(G) \cap E(G') = \varnothing$
   - $E(G) \cup E(G') = \{ \{v,w\} \mid v,w \in V \wedge v \neq w \}$
3) A **walk** from vertices v_0 to v_n of a graph G is a finite alternating sequence $v_0 e_1 v_1 e_2 \ldots v_{n-1} e_n v_n$ of adjacent vertices and edges of G.
4) A graph *G* is **connected** iff given any two distinct vertices *v* and *w* of G, there is a walk from *v* to *w*.